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Integrales trigonométricas

Reac.

$\int sen^2(x)\cos^2(x)\,dx$

Manipular integrando.

$sen^2(x)\cos^2(x)$

$sen^2(\theta)\cos^2(\theta) = \dfrac{1-\cos(4\theta)}{8}$

$\theta = x$., $4\theta = 4x$

$\dfrac{1-\cos(4x)}{8}$

$\int \dfrac{1-\cos(4x)}{8}\,dx$

$\dfrac{1}{8}\int 1-\cos(4x)\,dx$

$\dfrac{1}{8}\Big[\underbrace{\int 1\,dx}_{A} - \underbrace{\int \cos(4x)\,dx}_{B}\Big]$

Para A:

$\int 1\,dx = x + C$

$\int dx = x + C$

Para B:

$\int \cos(4x)\,dx$

$\int \cos(kx) = \dfrac{1}{k}sen(kx) + C$

$\dfrac{1}{4}sen(4x) + C$

$\dfrac{1}{8}\Big[(x) - \Big(\dfrac{1}{4}sen(4x)\Big)\Big] + C$

$\therefore F(x) = \dfrac{1}{8}x - \dfrac{1}{32}sen(4x) + C$

Reac 2.

$\int sen^4(\theta)\cos^3(\theta)\,d\theta$

Manipular el integrando

$sen^4(\theta)\cos^3(\theta)$

$sen^m(\theta)\cos^n(\theta)$

'n' es non y positivo entonces se factoriza el coseno.

$a^n \cdot a^m = a^{n+m}$

$a^1 = a$

$sen^4(\theta)\cos^2(\theta)\cos(\theta)$

$sen^2(\theta) + \cos^2(\theta) = 1$

$\cos^2(\theta) = -sen^2(\theta) + 1$

$sen^4(\theta)(-sen^2(\theta)+1)\cos(\theta)$

$sen^4(\theta)\cos(\theta)(-sen^2(\theta)+1)$

$a^n \cdot a^m = a^{n+m}$

$-sen^6(\theta)\cos(\theta) + sen^4(\theta)\cos(\theta)$

$\int -sen^6(\theta)\cos(\theta) + sen^4(\theta)\cos(\theta)\,d\theta$

$-\int sen^6(\theta)\cos(\theta)\,d\theta + \int sen^4(\theta)\cos(\theta)\,d\theta$

$\int f(g(x))g'(x)\,dx = \int f(u)\,du$., $u = g(x)$

Cambio de variable.

$u = sen(\theta)$., $f = u^6$ | $u = sen(x)$., $f = u^4$

$u' = \cos(\theta)$

$\Rightarrow \dfrac{du}{dx} = \dfrac{\cos(\theta)}{1}$

$dx = \dfrac{du}{\cos(\theta)}$

$-\int u^6 \cancel{\cos(\theta)}\dfrac{du}{\cancel{\cos(\theta)}} + \int u^4 \cancel{\cos(\theta)}\dfrac{du}{\cancel{\cos(\theta)}}$

$-\int u^6\,du + \int u^4\,du$

$\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$

$-\Big[\dfrac{u^7}{7}\Big] + \Big[\dfrac{u^5}{5}\Big] + C$

$-\dfrac{1}{7}u^7 + \dfrac{1}{5}u^5 + C$

$-\dfrac{1}{7}(sen(\theta))^7 + \dfrac{1}{5}(sen(\theta))^5 + C$

$\therefore F(\theta) = \dfrac{1}{5}sen^5(\theta) - \dfrac{1}{7}sen^7(\theta) + C$

Reac 3.

$$\int \tan^2(\alpha)\sec^4(\alpha)\,d\alpha$$

Manipular el integrando.

$\tan^2(\alpha)\sec^4(\alpha)$

$\tan^m(\theta)\sec^n(\theta)$

'n' es par y positiva, se factoriza $\sec^n(\theta)$ y se aplica:

$\sec^2(\theta) = 1+\tan^2(\theta)$

$\tan^2(\alpha)\sec^2(\alpha)\sec^2(\alpha)$

$\tan^2(\alpha)(1+\tan^2(\alpha))\sec^2(\alpha)$

$(\tan^2(\alpha)+\tan^4(\alpha))\sec^2(\alpha)$

$\sec^2(\alpha)\tan^2(\alpha)+\sec^2(\alpha)\tan^4(\alpha)$

$$\int \sec^2(\alpha)\tan^2(\alpha)+\sec^2(\alpha)\tan^4(\alpha)\,d\alpha$$

$$\int \sec^2(\alpha)\tan^2(\alpha)\,d\alpha+\int \sec^2(\alpha)\tan^4(\alpha)\,d\alpha$$

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du.,\ u=g(x)$$

Cambio de variable.

$u=\tan(\alpha)$, $f=u^2$ | $u=\tan(\alpha)$, $f=u^4$

$u' = \sec^2(\alpha)$

$\rightarrow \dfrac{du}{d\alpha} = \dfrac{\sec^2(\alpha)}{1}$

$d\alpha = \dfrac{du}{\sec^2(\alpha)}$

$$\int \cancel{\sec^2(\alpha)}\,u^2\frac{du}{\cancel{\sec^2(\alpha)}}+\int \cancel{\sec^2(\alpha)}\,u^4\frac{du}{\cancel{\sec^2(\alpha)}}$$

$$\int u^2\,du+\int u^4\,du$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+c$$

$$\frac{u^3}{3}+\frac{u^5}{5}+c$$

$$\frac{1}{3}(\tan(\alpha))^3+\frac{1}{4}(\tan(\alpha))^4+c$$

$$\therefore F(\alpha)=\frac{1}{3}\tan^3(\alpha)+\frac{1}{5}\tan^5(\alpha)+c$$

Reac 4.

$$\int \sec^5(x)\tan^3(x)\,dx$$

Manipular integrando.

$\sec^5(x)\tan^3(x)$

$\tan^m(\theta)\sec^n(\theta)$

'm' es non y positivo, factorizar ambas expresiones y aplicar:

$\tan^2(\theta) = \sec^2(\theta)-1$

$\sec^4(x)\sec(x)\tan^2(x)\tan(x)$

$\sec^4(x)\tan^2(x)\sec(x)\tan(x)$

$\tan^2(\theta) = \sec^2(\theta)-1$

$\sec^4(x)(\sec^2(x)-1)\sec(x)\tan(x)$

$(\sec^6(x)-\sec^4(x))(\sec(x)\tan(x))$

$\sec^6(x)\sec(x)\tan(x)-\sec^4(x)\sec(x)\tan(x)$

$$\int \sec^6(x)\sec(x)\tan(x)-\sec^4(x)\sec(x)\tan(x)\,dx$$

$$\int \sec^6(x)\sec(x)\tan(x)\,dx-\int \sec^4(x)\sec(x)\tan(x)\,dx$$

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du.,\ u=g(x)$$

Cambio de variable.

$u=\sec(x)$, $f=u^6$ | $u=\sec(x)$, $f=u^4$

$u' = \sec(x)\tan(x)$

$\rightarrow \dfrac{du}{dx} = \dfrac{\sec(x)\tan(x)}{1}$

$dx = \dfrac{du}{\sec(x)\tan(x)}$

$$\int u^6\,\cancel{\sec(x)\tan(x)}\frac{du}{\cancel{\sec(x)\tan(x)}}\ \ldots$$

$$\ldots-\int u^4\,\cancel{\sec(x)\tan(x)}\frac{du}{\cancel{\sec(x)\tan(x)}}$$

$$\int u^6\,du-\int u^4\,du$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+c$$

$$\frac{u^7}{7} - \frac{u^5}{5} + c$$

$$\frac{1}{7}(\sec(x))^7 - \frac{1}{5}(\sec(x))^5 + c$$

$$\therefore F(x) = \frac{1}{7}\sec^7(x) - \frac{1}{5}\sec^5(x) + c$$